NAME: SOLUTIONS

CONTINUING EDUCATION
MATHEMATICS DEPARTMENT

PART A: Fill in the blanks. Write your answer in the space provided.

1) Given points A(-4 , 5) and B(2 , -3), determine:

a) $\vec{BA} =$ $a-b = (-4-2, 5-(-3))$ $(-6, 8)$

b) $\left|\vec{AB}\right| = \sqrt{(2-(-4))^2 + (-3-5)^2} = \sqrt{36+64}$ 10

2) Suppose $\vec{a}=[1,2,-2]$, $\vec{b}=[2,-1,3]$ and $\vec{c}=[-2,-1,3]$. Determine:

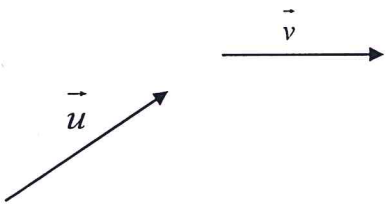
a) $\vec{a}\bullet\left(\vec{b}+\vec{c}\right) =$ $(1,2,-2)\cdot(0,-2,6) = 0-4-12$ -16

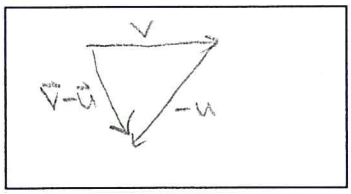
b) vector perpendicular to $\vec{a}$ and $\vec{b}$ = $\begin{vmatrix} 1 & 2 & -2 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \end{vmatrix}$ $(4,-7,-5)$

3) If $\vec{AB}=[1,2,-4]$ and point B has coordinates $B\left(-4,\frac{1}{2},-2\right)$ determine the coordinates of A. $-4-x=1$ $-2-z=-4$ $\frac{1}{2}-y=2$ $\left(-5,-\frac{3}{2},2\right)$

4) If $\vec{a}=[2,-3m,-1]$ and $\vec{b}=[4,-1,m]$, for what value of m is $\vec{a}$ perpendicular to $\vec{b}$. $a\cdot b=0$ $(8)+3m-m=0$ $8+2m=0$ $m=-\frac{8}{2}$ $m=-4$

5) Given the following vectors, draw the vector that represents $\vec{v}-\vec{u}$.





6) If $\vec{a}=[4,2,1]$ and $\vec{b}=[-2,-1,3]$, determine $\vec{b}\times\vec{a}$ $(-7, 14, 0)$

7) Write the vector equation for the line given by the symmetric equations $\frac{x-4}{3}=\frac{y+1}{-2}=\frac{z-7}{1}$ $\vec{r}=(4,-1,7)+t(3,-2,1)$

$x-4=3t$ $y+1=-2t$ $z-7=t$

$x=4+3t$ $y=-1-2t$ $z=7+t$

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8) Write the parametric equations for the line passing through the point (-2 , 4) and perpendicular to $\vec{m} = [3,-1]$ $m_\perp = (1,3)$

$x = -2 + t$
$y = 4 + 3t$

9) State the normal for the plane $3 - 5y + 7z + 2x = 0$

$n = (2, -5, 7)$

10) When is $\vec{u} \cdot \vec{v}$ negative?

$90 < \theta < 180°$

11) Name the vector that is equivalent to $\vec{BC} - \vec{DC} - \vec{ED}$

$\overline{BE}$

$BC + CD + DE$

12) Find the directional cosines of vector $[2,-4,1]$

To the nearest degree.

$\cos\alpha = \frac{2}{\sqrt{2^2+(-4)^2+1^2}} = \frac{2}{\sqrt{21}}$

$\alpha = 64°$

$\beta = 151°$

$\chi = 77°$

Part B: Short Answer Questions

1) Calculate the angles ***to the nearest degree*** of triangle ABC with vertices A (5 , -3) , B (1 , 8) and C (1 , -2). What type of triangle is ABC?

$\overline{AB} = (-4, 11)$
$\overline{AC} = (-4, 1)$

$\overline{AC} \cdot \overline{AB} = (-4)(-4) + (11)(1)$
$= 16 + 11$
$= 27$

$27 = (\sqrt{(-4)^2+11^2})(\sqrt{(-4)^2+1^2})\cos A$

$\frac{27}{(\sqrt{137})(\sqrt{17})} = \cos A$

$A = 60°$

$\overline{BC} = (0, -10)$
$\overline{BA} = (4, -11)$

$\overline{BC} \cdot \overline{BA} = (0)(4) + (-10)(-11)$
$= 110$

$110 = (\sqrt{(-10)^2})(\sqrt{4^2+(-11)^2})\cos B$

$\frac{110}{(10)(\sqrt{137})} = \cos B$

$B = 20°$

$|\overline{AB}| = \sqrt{137}$
$|\overline{BC}| = 10$
$|\overline{CA}| = \sqrt{17}$

$C = 180 - 60 - 20$
$= 100°$

$\therefore \Delta ABC$ is a scalene, obtuse triangle

2) Determine if the points A (-2 , 2 , 7) , B (6 , -2 , 1) and C (-6 , 4 , 10) are collinear.

$\overrightarrow{AB} = (8, -4, -6)$

$\overrightarrow{BC} = (-12, 6, 9)$

$\overrightarrow{AB} = K\,\overrightarrow{BC}$

$(8, -4, -6) = K(-12, 6, 9)$

$8 = -12K$

$\frac{8}{-12} = K$

$-\frac{2}{3} = K$

check

$-4 = 6K$

LS -4 RS $= 6\left(-\frac{2}{3}\right)$

$= -\frac{12}{3}$

$= -4$

$-6 = 9K$

LS -6 RS $= 9\left(-\frac{2}{3}\right)$

$= -18/3$

$= -6$

$\therefore$ points are collinear

$\overrightarrow{AC} = (-4, 2, 3)$

$\overrightarrow{AB} + \overrightarrow{BC}$

$= (8, -4, -6) + (-12, 6, 9)$

$= (-4, 2, 3)$

$= \overrightarrow{AC}$

$\therefore$ collinear.

3) Calculate the area of the parallelogram ***to two decimal places*** determined by each pair of vectors.

$\vec{a} = [-1, 1, 2]$ and $\vec{b} = [0, 3, 4]$

$a \times b = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ 0 & 3 & 4 \end{vmatrix}$

$= (4 - 6, 0 - (-4), -3 - 0)$

$= (-2, 4, -3)$

$|\overrightarrow{a \times b}| = \sqrt{(-2)^2 + 4^2 + (-3)^2}$

$= \sqrt{4 + 16 + 9}$

$= \sqrt{29}$

$\doteq 5.39$ units^2

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4) Find parametric equations of the line of intersection of the following planes.

$\pi_1 : 3x - y + 4z = 7$ $\quad \vec{n}_1 = (3, -1, 4)$

$\pi_2 : x + y - 2z + 5 = 0$ $\quad \vec{n}_2 = (1, 1, -2)$

$\vec{m} = \vec{n}_1 \times \vec{n}_2$

$= \begin{vmatrix} 3 & -1 & 4 & 3 & -1 \\ 1 & 1 & -2 & 1 & 1 \end{vmatrix}$

$= (2-4, 4+6, 3+1)$

$= (-2, 10, 4)$

$= (-1, 5, 2)$

Find a pt

let $z = 0$

$3x - y = 7$ ①

$x + y = -5$ ②

① + ②, $4x = 2$

$x = \frac{2}{4}$

$y = 3x - 7$

$= 3(\frac{1}{2}) - 7$

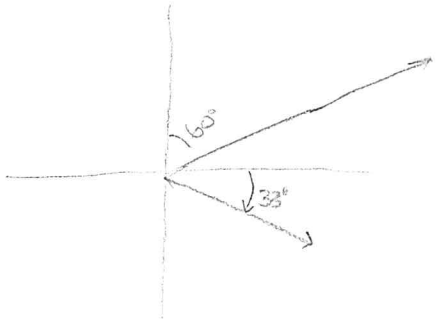
$= \frac{3}{2} - \frac{14}{2}$

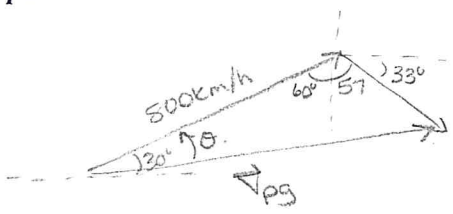
$= \frac{-11}{2}$

∴ pt is $(\frac{1}{2}, \frac{-11}{2}, 0)$

$\vec{r} = (\frac{1}{2}, \frac{-11}{2}, 0) + s(-1, 5, 2)$

5) A plane is flying on a bearing of 60^0 at a constant speed of 800 km/h. If the velocity of the wind is 120 km/h on a bearing of 123^0, what is the velocity of the plane relative to the ground? ***Round final answers to one decimal place.***





$|\vec{v}_{pg}|^2 = 800^2 + 120^2 - 2(800)(120)\cos 117°$

$= 741566$

$|\vec{v}_{pg}| = 861 \text{ km/h}$

$\frac{\sin\theta}{120} = \frac{\sin 117}{861.14}$

$\theta = 7°$

$\vec{v}_{pg} = 861 \frac{\text{km}}{\text{h}} [E\ 23°\ N]$

or $= 861 \frac{\text{km}}{\text{h}}$ on bearing 67°

Part C: **Problem Solving**

Show all your work, providing neat and clear solutions in the space provided.

1) Determine the scalar equation of the plane that contains the intersecting lines;

$$\frac{x-2}{1}=\frac{y}{2}=\frac{z+3}{3} \quad \textit{and} \quad \frac{x-2}{-3}=\frac{y}{4}=\frac{z+3}{2}$$

$\vec{m} = (1,2,3)$ $\vec{m} = (-3,4,2)$

$\vec{n} = \vec{m} \times \vec{m}$

$= \begin{vmatrix} 2 & 3 & 1 & 2 \\ -3 & 4 & 2 & -3 \end{vmatrix}$

$= (4-12, -9-2, 4-(-6))$

$= (-8, -11, 10)$

$\pi: \quad -8x - 11y + 10z + D = 0$

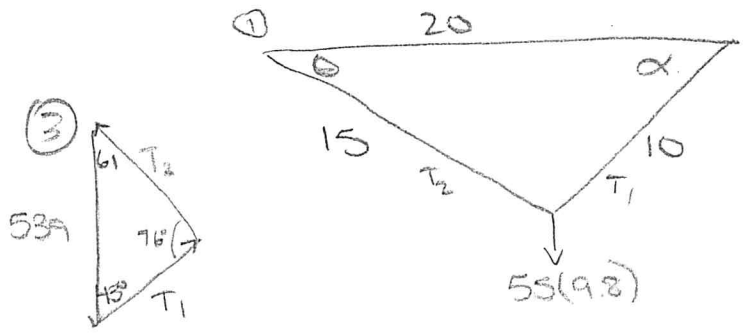
pt $(2,0,-3)$

$-8(2) - 11(0) + 10(-3) + D = 0$

$-16 - 30 + D = 0$

$D = 46$

2) Enrico performs a high wire act at the circus. During his act, he stands on a 25 m wire, 10 m from one platform and 15 m from a second platform. The distance between the two platforms is 20 m. Find the tensions being exerted in each of the two sections of the wire if Enrico has a mass of 55 Kg. [N.B. Assume that a 1 Kg mass exerts a force of 9.8 N].
Round all answers to one decimal place.



② $10^2 = 15^2 + 20^2 - 2(15)(20)\cos\theta$

$\frac{-525}{-600} = \cos\theta$

$\theta = 29°$

$\frac{\sin\alpha}{15} = \frac{\sin\theta}{10}$

$\alpha = 47°$

③ $\frac{T_1}{\sin 61°} = \frac{539}{\sin 76°}$ $\quad$ $\frac{T_2}{\sin 43°} = \frac{539}{\sin 76}$

$T_1 = 485.9\,N$ $\quad$ $T_2 = 378.9\,N$

3) Give a geometric interpretation of the following system of equations. Show your work.

π_1: $2x + 3y - 5z = 9$
π_2: $5x - y + 2z = -3$
π_3: $-x + 7y - 12z = 21$

$\vec{n}_1 = (2, 3, -5)$
$\vec{n}_2 = (5, -1, 2)$
$\vec{n}_3 = (-1, 7, -12)$

$n_1 \neq kn_2$
$n_1 \neq kn_3$
$n_2 \neq kn_3$
} not parallel.

Not perpendicular $\vec{n}_1 \cdot \vec{n}_2 = 10 - 3 - 10 = -3$ $\quad$ $n_2 \cdot n_3 = -5 - 7 - 24 = -36$ $\quad$ $n_1 \cdot n_3 = -2 + 21 + 60 = 79$

$n_1 \times n_2 = (1, -29, -17)$ $\quad$ $\theta_{12} = 95°$
$n_1 \times n_3 = (-1, 29, 17)$ $\quad$ $\theta_{13} = 23°$
$n_2 \times n_3 = (-1, 29, 17)$ $\quad$ $\theta_{23} = 62°$

Planes intersect at a line

$x = t$
$y = 3 - 29t$
$z = -17t$